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Null Geodesics in Brane World Universe

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Abstract

We study null bulk geodesic motion in the brane world cosmology in the RS2 scenario and in the static universe in the bulk of the charged topological AdS black hole. We obtain equations describing the null bulk geodesic motion as observed in one lower dimensions. We find that the null geodesic motion in the bulk of the brane world cosmology in the RS2 scenario is observed to be under the additional influence of extra non-gravitational force by the observer on the three-brane, if the brane universe does not possess the $\mathbf{Z}_2$ symmetry. As for the null geodesic motion in the static universe in the bulk of the charged AdS black hole, the extra force is realized even when the brane universe has the $\mathbf{Z}_2$ symmetry.

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1 Introduction

Recently, there has been active interest in the possibility that our four-dimensional universe might be embedded in higher-dimensional bulk spacetime having warped product structure [1, 2], as such possibility may provide possible attractive solutions to the hierarchy and the cosmological constant problems. Unlike the case of the conventional compactification with compact extra space, the bulk metrics of the brane world scenarios have nontrivial dependence on the extra spatial coordinates. This fact implies that the geodesic motion in such bulk spacetime is observed in the embedded lower-dimensional spacetime as being under the influence of the extra non-gravitational force, called the fifth force [3, 4, 5, 6, 7, 8, 9, 10, 11, 12], if the velocity of the test particle has non-zero components along the extra spatial directions. In our previous work [13], we analyzed bulk geodesic motion in the general symmetrically warped spacetime of codimension one and identified the extra force. We found that particle motion due to such force violates conventional laws of particle mechanics in lower dimensions, thereby hinting at the higher-dimensional origin of the embedded spacetime.

In this paper, we study null geodesic motion in the bulk of various brane universe models. In brane world scenarios, gravitons are assumed to propagate freely in the bulk, whereas all the matter fields are assumed to be confined on the brane. Therefore, it is expected that gravitons are observed to be under the influence of the extra force from the perspective of an observer living on the brane. Furthermore, gravitons with nontrivial motion in the extra spatial dimension are observed to be massive from the perspective of the lower-dimensional observer. This can be understood from the fact that the momentum component along the extra spatial dimension can be interpreted as being related to mass in lower dimensions. Furthermore, since the extra force observed in lower dimensions has nonzero component along the direction parallel to the four-velocity, the mass is observed to vary with time. These facts may be used to test whether our universe is described by the brane world scenario.

The paper is organized as follows. In section 2, we consider the null geodesic motion in the bulk of the brane world cosmology in the Randall-Sundrum (RS) scenario with one positive tension brane and noncompact extra space [2], i.e., the RS2 scenario. We find that the extra force is zero (i.e., the null bulk geodesic motion is observed as the timelike geodesic motion on the brane) in the $\mathbf{Z}_2$ symmetric brane universe. The extra force turns out to be proportional to the constant parametrizing the extend to which $\mathbf{Z}_2$ symmetry is broken. In section 3, we consider the $\mathbf{Z}_2$ symmetric static brane universe in the bulk of the charged topological AdS black hole. Even with the $\mathbf{Z}_2$ symmetry, the extra force is observed in lower dimensions.

2 Null Bulk Geodesics in the RS2 Brane World Cosmology

In this section, we study the null bulk geodesic motion in the brane world cosmology in the RS2 scenario [2]. The action for the model is given by

$$S=\int d^5x\sqrt{-G}\left[\frac{1}{2\kappa^2}\mathcal{R}-\Lambda\right]+\int d^4x\sqrt{-g}\left[\mathcal{L}_{\rm mat}-\sigma\right], \tag{1}$$

where $\mathcal{L}_{\rm mat}$ is the Lagrangian for the matter fields on the brane, κ is the five-dimensional gravitational constant, Λ is the bulk cosmological constant, and σ is the tension of the brane assumed to be located at the origin $y=0$ of the extra special coordinate y. The metric $g_{\mu\nu}$ on the brane is given in terms of the bulk metric G_{MN} by $g_{\mu\nu}(x^\rho)=G_{\mu\nu}(x^\rho,0)$. The general bulk metric ansatz for the brane world cosmology with stabilized extra spatial dimension is

$$G_{MN}dx^Mdx^N=-n^2(t,y)dt^2+a^2(t,y)\gamma_{ij}dx^idx^j+dy^2, \tag{2}$$

where γ_{ij} is the metric for the maximally symmetric three-dimensional space given by

$$\gamma_{ij}dx^idx^j=\left(1+\tfrac{k}{4}\delta_{mn}x^mx^n\right)^{-2}\delta_{ij}dx^idx^j. \tag{3}$$

The curvature parameter k is -1, 0, 1, respectively for the three-dimensional space with the negative, zero and positive spatial curvature. For the phenomenological relevance, we consider the flat universe case ($k=0$), only, in this paper.

We study the geodesic motion in the bulk spacetime with the metric given by Eq. (2). The geodesic motion of a test particle in the bulk spacetime is described by the geodesic equations

$$\frac{d^2x^R}{d\lambda^2}+\hat{\Gamma}^R_{MN}\frac{dx^M}{d\lambda}\frac{dx^N}{d\lambda}=0, \tag{4}$$

which take the following forms after the explicit expression (2) for the bulk metric with $k=0$ is substituted:

$$\frac{d^2t}{d\lambda^2}+\frac{\dot n}{n}\left(\frac{dt}{d\lambda}\right)^2+2\frac{n'}{n}\frac{dt}{d\lambda}\frac{dy}{d\lambda}+\frac{a\dot a}{n^2}\sum_j\left(\frac{dx^j}{d\lambda}\right)^2=0, \tag{5}$$

$$\frac{d^2x^i}{d\lambda^2}+2\frac{\dot a}{a}\frac{dt}{d\lambda}\frac{dx^i}{d\lambda}+2\frac{a'}{a}\frac{dy}{d\lambda}\frac{dx^i}{d\lambda}=0, \tag{6}$$

$$\frac{d^2y}{d\lambda^2}+nn'\left(\frac{dt}{d\lambda}\right)^2-aa'\sum_j\left(\frac{dx^j}{d\lambda}\right)^2=0. \tag{7}$$

Here, $\hat{\Gamma}^R_{MN}$ is the Christoffel symbol (of the second kind) for the bulk metric G_{MN} and λ is an affine parameter for the geodesic path $x^M(\lambda)$. The affine parameter λ is defined by the following metric compatibility condition along the geodesic path:

$$-\epsilon_5 = G_{MN}\frac{dx^M}{d\lambda}\frac{dx^N}{d\lambda} = -n^2\left(\frac{dt}{d\lambda}\right)^2 + a^2\delta_{ij}\frac{dx^i}{d\lambda}\frac{dx^j}{d\lambda} + \left(\frac{dy}{d\lambda}\right)^2, \tag{8}$$

where $\epsilon_5 = 0, 1$ respectively for a massless test particle (i.e., a null geodesic motion) and a massive test particle (i.e., a timelike geodesic motion). Since it is generally assumed in the brane world scenarios that only gravitons can propagate freely in the bulk, in this paper we consider the null bulk geodesic motion (i.e., the $\epsilon_5 = 0$ case), only.

We rewrite the bulk geodesic equations (5-7) in terms of quantities of four-dimensional spacetime on the hypersurface $y = const$, for the purpose of studying particle dynamics as observed in one lower dimensions. Since the metric on the hypersurface is given by

$$g_{\mu\nu}dx^\mu dx^\nu = -n^2dt^2 + a^2\delta_{ij}dx^i dx^j, \tag{9}$$

the affine parameter $\tilde{\lambda}$ for the motion observed on the hypersurface is defined by

$$-\epsilon_4 = g_{\mu\nu}\frac{dx^\mu}{d\tilde{\lambda}}\frac{dx^\nu}{d\tilde{\lambda}} = -n^2\left(\frac{dt}{d\tilde{\lambda}}\right)^2 + a^2\delta_{ij}\frac{dx^i}{d\tilde{\lambda}}\frac{dx^j}{d\tilde{\lambda}}, \tag{10}$$

where $\epsilon_4 = -1, 0, 1$ respectively for a spacelike, a lightlike and a timelike motions observed on the three-brane. The $y = 0$ case, for which $n = 1$ and $a = a_0(t) \equiv a(t, 0)$, corresponds to the metric compatibility condition for the motion observed on the three-brane. We assume that the affine parameter $\tilde{\lambda}$ for the motion observed on the hypersurface is a smooth function of the affine parameter λ for the bulk geodesic motion: $\tilde{\lambda} = f(\lambda)$. Then, the consistency equation (8) for the bulk geodesic motion can be rewritten in terms of the new parameter $\tilde{\lambda}$ as

$$-n^2\left(\frac{dt}{d\tilde{\lambda}}\right)^2 + a^2\delta_{ij}\frac{dx^i}{d\tilde{\lambda}}\frac{dx^j}{d\tilde{\lambda}} = -\left[\epsilon_5\left(\frac{d\lambda}{d\tilde{\lambda}}\right)^2 + \left(\frac{dy}{d\tilde{\lambda}}\right)^2\right]. \tag{11}$$

For gravitons ($\epsilon_5 = 0$), which freely move along the extra spatial direction (i.e., $\frac{dy}{d\lambda} \neq 0$), the bulk geodesic motion can be observed only as timelike ($\epsilon_4 = 1$) on the hypersurface. Namely, gravitons are observed to be massive by observers living on the hypersurface. The new parameter $\tilde{\lambda}$ is an affine parameter for such motion, if the following is satisfied:

$$\left(\frac{dy}{d\tilde{\lambda}}\right)^2 = 1, \tag{12}$$

for which the RHS of Eq. (11) becomes -1. To express the bulk geodesic equations (5-7) in terms of the new parameter $\tilde{\lambda}$, we obtain the relation between the two parameters λ and $\tilde{\lambda}$ by making use of Eqs. (7,12). The relation is given by

$$\left(\frac{d\tilde{\lambda}}{d\lambda}\right)^{-1}\frac{d}{d\tilde{\lambda}}\left(\frac{d\tilde{\lambda}}{d\lambda}\right) = -nn'\left(\frac{dt}{d\tilde{\lambda}}\right)^2 + aa'\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2. \tag{13}$$

Making use of Eqs. (12,13), we reexpress the t- and x^i-component bulk geodesic equations (5,6) in terms of the new parameter $\tilde{\lambda}$ as follows

$$\frac{d^2t}{d\tilde{\lambda}^2}+\frac{\dot{n}}{n}\left(\frac{dt}{d\tilde{\lambda}}\right)^2+\frac{a\dot{a}}{n^2}\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2=\left[nn'\left(\frac{dt}{d\tilde{\lambda}}\right)^2-aa'\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2-2\frac{n'}{n}\right]\frac{dt}{d\tilde{\lambda}}, \quad (14)$$

$$\frac{d^2x^i}{d\tilde{\lambda}^2}=\left[nn'\left(\frac{dt}{d\tilde{\lambda}}\right)^2-aa'\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2-2\frac{a'}{a}\right]\frac{dx^i}{d\tilde{\lambda}}. \quad (15)$$

Note, the LHS's of these equations have the forms $\frac{d^2x^\rho}{d\tilde{\lambda}^2}+\Gamma^\rho_{\mu\nu}\frac{dx^\mu}{d\tilde{\lambda}}\frac{dx^\nu}{d\tilde{\lambda}}$, where $\Gamma^\rho_{\mu\nu}$ is the Christoffel symbol for the metric $g_{\mu\nu}dx^\mu dx^\nu=-n^2(t,y_0)dt^2+a^2(t,y_0)\delta_{ij}dx^idx^j$ on the hypersurface $y=y_0=const.$ The RHS's are identified as the time and the spatial components of the four-acceleration vector A^μ of the particle due to non-gravitational force. So, we see that the null bulk geodesic motion is observed on the hypersurface to be the timelike motion under the additional influence of extra non-gravitational force.

The observer on the hypersurface will find something unusual about the four-acceleration vector. Namely, the four-acceleration vector has nonzero component parallel to the four-velocity:

$$g_{\mu\nu}A^\mu\frac{dx^\nu}{d\tilde{\lambda}}=\left[2-n^2\left(\frac{dt}{d\tilde{\lambda}}\right)^2+a^2\sum_i\left(\frac{dx^i}{d\tilde{\lambda}}\right)^2\right]\left[nn'\left(\frac{dt}{d\tilde{\lambda}}\right)^2-aa'\sum_i\left(\frac{dx^i}{d\tilde{\lambda}}\right)^2\right], \quad (16)$$

contrary to the conventional laws of particle mechanics. An observer who is unaware of the existence of extra dimensions will interpret such abnormal four-acceleration component as being due to the variation of the particle mass [8, 10] in the following way. Taking into account the possibility of time-variable proper mass m of a particle, we write the force law for the particle motion in curved space time as

$$F^\mu=\frac{Dp^\mu}{d\tilde{\lambda}}=\frac{dm}{d\tilde{\lambda}}\frac{dx^\mu}{d\tilde{\lambda}}+mA^\mu, \quad (17)$$

where $p^\mu\equiv m\frac{dx^\mu}{d\tilde{\lambda}}$ is the four-momentum of the particle. The four-dimensional observer will assume that $g_{\mu\nu}F^\mu\frac{dx^\nu}{d\tilde{\lambda}}=0$, since all the known forces in nature act perpendicularly to the four-velocity $\frac{dx^\mu}{d\tilde{\lambda}}$ of a particle. So, the four-dimensional observer will conclude that the mass of the particle varies with time as

$$\frac{1}{m}\frac{dm}{d\tilde{\lambda}}=g_{\mu\nu}A^\mu\frac{dx^\nu}{d\tilde{\lambda}}, \quad (18)$$

which says that the proper mass of a particle varies with time when its four-acceleration vector has nonzero component parallel to its four-velocity vector. As was pointed out in Ref. [11], the component of the four-acceleration vector parallel to the four-velocity vector can be removed through non-affine transformation of the parameter $\tilde{\lambda}$, and therefore the time variation of the particle mass may be regarded as an artifact of choosing wrong parameter $\tilde{\lambda}$ for the particle motion. In fact, there is an ambiguity of choosing the right parameter for the particle motion. However, since the metric on the hypersurface is given by $g_{\mu\nu}$, an observer, who is unaware of the extra dimension, will choose the parameter $\tilde{\lambda}$ satisfying $g_{\mu\nu}\frac{dx^\mu}{d\tilde{\lambda}}\frac{dx^\nu}{d\tilde{\lambda}}=-1$ as the natural parameter describing a timelike motion observed on the hypersurface.

We now study the bulk geodesic motion as observed on the three-brane (located at $y=0$). Since the first y derivatives of metric components are discontinuous at $y=0$ due to the δ-function singularity there, Eqs. (14,15) are not well-defined at $y=0$. Nevertheless, we can obtain the effective equations on the three-brane by taking the mean values of Eqs. (14, 15) across $y=0$ and applying the following boundary conditions on the first derivatives of the metric components:

$$[a']_0 = -\frac{\kappa^2}{3}a_0\left(\sigma+\varrho\right), \tag{19}$$

$$[n']_0 = -\frac{\kappa^2}{3}\left(\sigma-3\wp-2\varrho\right), \tag{20}$$

where ϱ and $\wp$ are the mass density and the pressure of the brane matter fields, and $[F]_0\equiv F(0^+)-F(0^-)$ denotes the jump of $F(y)$ across $y=0$. We define the mean value of a function F across $y=0$ as $\sharp F\sharp\equiv[F(0^+)+F(0^-)]/2$. When the brane universe is invariant under the $\mathbf{Z}_2$ transformation, $y\to -y$, then the first derivatives of the metric components satisfy $a'(0^+)=-a'(0^-)$ and $n'(0^+)=-n'(0^-)$. So, the mean values of Eqs. (14,15) across $y=0$ respectively take the forms:

$$\frac{d^2t}{d\tilde{\lambda}^2}+a_0\dot{a}_0\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2=0, \tag{21}$$

$$\frac{d^2x^i}{d\tilde{\lambda}^2}=0, \tag{22}$$

where we made use of the fact that the time coordinate t is defined such that $n=1$ at $y=0$. These are just geodesic equations for a test particle moving in the gravitational field $g_{\mu\nu}dx^\mu dx^\nu = -dt^2+a_0^2(t)\delta_{ij}dx^idx^j$. Namely, the null bulk geodesic motion is observed on the three-brane as the timelike geodesic motion. Next, we consider the brane universe without the $\mathbf{Z}_2$ symmetry. Without the $\mathbf{Z}_2$ symmetry, the first derivative of a around $y=0$ satisfies

$$a'(0^+)=-a'(0^-)+d(t), \tag{23}$$

where $d(t)$ is an arbitrary function of t. By applying the boundary condition (19) and taking the jump of the (y,y)-component Einstein equation, $d(t)$ can be determined to take the following form [14]:

$$d(t)=\frac{2F}{(\sigma+\varrho)a_0^3}, \tag{24}$$

where an integration constant F parametrizes the extent to which the $\mathbf{Z}_2$ symmetry is broken. The mean values of Eqs. (14, 15) then respectively take the following forms:

$$\frac{d^2t}{d\tilde{\lambda}^2}+a_0\dot{a}_0\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2=-\frac{F}{(\sigma+\varrho)a_0^2}\frac{dt}{d\tilde{\lambda}}\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2, \tag{25}$$

$$\frac{d^2x^i}{d\tilde{\lambda}^2}=-\frac{F}{\sigma+\varrho}\left[\frac{1}{a_0^2}\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2+\frac{2}{a_0^4}\right]\frac{dx^i}{d\tilde{\lambda}}. \tag{26}$$

So, the null bulk geodesic motion is observed on the three-brane of asymmetric brane universe to be under the additional influence of non-gravitational force. The four-acceleration vector A^μ has non-zero component parallel to the four-velocity and therefore the proper mass of the particle is observed to vary with time as

$$\frac{1}{m}\frac{dm}{d\tilde{\lambda}}=g_{\mu\nu}A^\mu\frac{dx^\nu}{d\tilde{\lambda}}=\frac{F}{(\sigma+\varrho)a_0^2}\left[\left(\frac{dt}{d\tilde{\lambda}}\right)^2-a_0^2\sum_i\left(\frac{dx^i}{d\tilde{\lambda}}\right)^2-2\right]\sum_i\left(\frac{dx^i}{d\tilde{\lambda}}\right)^2. \tag{27}$$

The four-acceleration of the particle and the time variation of the particle mass are proportional to the asymmetry parameter F, and therefore vanish in the limit of $F=0$, for which the brane world becomes $\mathbf{Z}_2$ symmetric. We expect that these results continue to hold for the brane world cosmology in the RS1 model [1]. Making use of these facts, we can experimentally determine whether our universe, assumed to be modeled by the RS scenario, is $\mathbf{Z}_2$ symmetric or not.

3 Null Bulk Geodesics in Static Universe in the bulk of Charged AdS Black Hole

In this section, we consider the brane universe in the bulk of the charged topological AdS black hole. The bulk metric has the following form:

$$\begin{aligned}G_{MN}dx^Mdx^N &= -h(y)dt^2+y^2\gamma_{ij}dx^idx^j+\frac{1}{h(y)}dy^2,\\ h(y) &= k-\frac{\mu}{y^2}+\frac{q^2}{y^4}+\frac{y^2}{l^2},\end{aligned} \tag{28}$$

where q and μ are respectively proportional to the charge and the ADM mass of the black hole, and l is the curvature radius of the background AdS spacetime. For the $k=0$ case, the bulk geodesic equations (4) take the following forms:

$$\frac{d^2t}{d\lambda^2}+\frac{h'}{h}\frac{dt}{d\lambda}\frac{dy}{d\lambda}=0, \tag{29}$$

$$\frac{d^2x^i}{d\lambda^2}+\frac{2}{y}\frac{dx^i}{d\lambda}\frac{dy}{d\lambda}=0, \tag{30}$$

$$\frac{d^2y}{d\lambda^2}+\frac{hh'}{2}\left(\frac{dt}{d\lambda}\right)^2-\frac{1}{2}\frac{h'}{h}\left(\frac{dy}{d\lambda}\right)^2-hy\sum_j\left(\frac{dx^j}{d\lambda}\right)^2=0, \tag{31}$$

and the metric compatibility condition along the geodesic path becomes

$$-\epsilon_5=-h\left(\frac{dt}{d\lambda}\right)^2+y^2\delta_{ij}\frac{dx^i}{d\lambda}\frac{dx^j}{d\lambda}+\frac{1}{h}\left(\frac{dy}{d\lambda}\right)^2. \tag{32}$$

In terms of a new parameter $\tilde{\lambda}=f(\lambda)$, the compatibility condition (32) is rewritten as

$$-h\left(\frac{dt}{d\tilde{\lambda}}\right)^2+y^2\delta_{ij}\frac{dx^i}{d\tilde{\lambda}}\frac{dx^j}{d\tilde{\lambda}}=-\left[\epsilon_5\left(\frac{d\lambda}{d\tilde{\lambda}}\right)^2+\frac{1}{h}\left(\frac{dy}{d\tilde{\lambda}}\right)^2\right]. \tag{33}$$

So, the null bulk geodesic motion ($\epsilon_5=0$) with $\frac{dy}{d\tilde{\lambda}}\neq 0$ is observed on the hypersurface $y=const$ as timelike. The parameter $\tilde{\lambda}$ is an affine parameter for such motion observed on the hypersurface, if the following is satisfied:

$$\left(\frac{dy}{d\tilde{\lambda}}\right)^2=h, \tag{34}$$

for which the RHS of Eq. (33) becomes -1. From Eqs. (31,34), we obtain the following relation between the parameters λ and $\tilde{\lambda}$:

$$\left(\frac{d\tilde{\lambda}}{d\lambda}\right)^{-1}\frac{d}{d\tilde{\lambda}}\left(\frac{d\tilde{\lambda}}{d\lambda}\right)=-\sqrt{h}\left[\frac{h'}{2}\left(\frac{dt}{d\tilde{\lambda}}\right)^2-y\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2\right]. \tag{35}$$

Making use of this relation, we can rewrite the t- and x^i-component geodesic equations Eqs. (29,30) in terms of the new parameter $\tilde{\lambda}$ as

$$\frac{d^2t}{d\tilde{\lambda}^2}=\sqrt{h}\left[\frac{h'}{2}\left(\frac{dt}{d\tilde{\lambda}}\right)^2-y\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2-\frac{h'}{h}\right]\frac{dt}{d\tilde{\lambda}}, \tag{36}$$

$$\frac{d^2x^i}{d\tilde{\lambda}^2}=\sqrt{h}\left[\frac{h'}{2}\left(\frac{dt}{d\tilde{\lambda}}\right)^2-y\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2-\frac{2}{y}\right]\frac{dx^i}{d\tilde{\lambda}}. \tag{37}$$

So, the null bulk geodesic motion is observed on the hypersurface to be a timelike motion under the additional influence of non-gravitational force. The four-acceleration vector has non-zero component parallel to the four-velocity vector, and therefore the proper mass of the particle is observed to vary with time:

$$\frac{1}{m}\frac{dm}{d\tilde{\lambda}}=g_{\mu\nu}A^\mu\frac{dx^\nu}{d\tilde{\lambda}}=\sqrt{h}\left[2-h\left(\frac{dt}{d\tilde{\lambda}}\right)^2+y^2\sum_i\left(\frac{dx^i}{d\tilde{\lambda}}\right)^2\right]\left[\frac{h'}{2}\left(\frac{dt}{d\tilde{\lambda}}\right)^2-y\sum_i\left(\frac{dx^i}{d\tilde{\lambda}}\right)^2\right]. \tag{38}$$

We construct the $\mathbf{Z}_2$ symmetric brane universe in this bulk background. We introduce a probe three-brane at $y = y_0$ and impose the $\mathbf{Z}_2$ symmetry under the transformation $y \to y_0/y^2$ by cutting off the spacetime in the region $y \geq y_0$ and then replacing it by a copy of the spacetime in the region $y \leq y_0$ transformed under $y \to y_0/y^2$ [15]. We introduce the brane matter with the mass density ϱ and the pressure $\wp$. In particular, we are interested in static brane configuration, where the three-brane remains fixed at $y = y_0$. Such configuration can be achieved by the brane matter satisfying [16]

$$6\sqrt{h(y_0)} = \kappa^2 \varrho y_0, \qquad 18h'(y_0) = -\kappa^4 \varrho(2\varrho + 3\wp)y_0. \tag{39}$$

Using the explicit expression for h with $k = 0$ and assuming the equation of state of the form $\wp = \omega\varrho$, we obtain the following relations among y_0 and the black hole parameters [16]:

$$\mu = 3\left(l^{-2} + \tfrac{1}{36}\kappa^4\omega\varrho^2\right) y_0^4, \qquad q^2 = 2\left(l^{-2} + \tfrac{1}{72}\kappa^4(1+3\omega)\varrho^2\right) y_0^6. \tag{40}$$

To obtain the equations for the null bulk geodesic motion as observed on the three-brane, we just substitute the expressions for $h(y_0)$, $h'(y_0)$ in Eq. (39) into Eqs. (36,37). The resulting expressions are given by

$$\frac{d^2t}{d\tilde{\lambda}^2} = -\frac{\kappa^2}{6}\varrho\left[\frac{\kappa^4}{36}(2+3\omega)\varrho^2 y_0^2\left(\frac{dt}{d\tilde{\lambda}}\right)^2 + y_0^2\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2 - 2(2+3\omega)\right]\frac{dt}{d\tilde{\lambda}}, \tag{41}$$

$$\frac{d^2x^i}{d\tilde{\lambda}^2} = -\frac{\kappa^2}{6}\varrho\left[\frac{\kappa^4}{36}(2+3\omega)\varrho^2 y_0^2\left(\frac{dt}{d\tilde{\lambda}}\right)^2 + y_0^2\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2 - 2\right]\frac{dx^i}{d\tilde{\lambda}}. \tag{42}$$

So, the time variation of the proper mass as observed on the brane is

$$\begin{aligned}\frac{1}{m}\frac{dm}{d\tilde{\lambda}} = g_{\mu\nu}A^\mu\frac{dx^\nu}{d\tilde{\lambda}} &= -\frac{\kappa^2}{6}\varrho\left[2 - \frac{\kappa^4}{36}\varrho^2 y_0^2\left(\frac{dt}{d\tilde{\lambda}}\right)^2 + y_0^2\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2\right] \\ &\times\left[\frac{\kappa^4}{36}(2+3\omega)\varrho^2 y_0^2\left(\frac{dt}{d\tilde{\lambda}}\right)^2 + y_0^2\sum_j\left(\frac{dx^j}{d\tilde{\lambda}}\right)^2\right].\end{aligned} \tag{43}$$

Unlike the case in the previous section, the observer on the three-brane will observe extra non-gravitational force, even when the brane universe possesses the $\mathbf{Z}_2$ symmetry. The strength of the extra force increases as mass density of brane matter increases.

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